Mutual Information Minimization for Side-Channel Attack Resistance via Optimal Noise Injection

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Abstract—Side-channel attacks (SCAs) pose a serious threat to system security by extracting secret keys through physical leakages such as power consumption, timing variations, and electromagnetic emissions. Among existing countermeasures, artificial noise injection is recognized as one of the most effective techniques. However, its high power consumption poses a major challenge for resource-constrained systems such as Internet of Things (IoT) devices, motivating the development of more efficient protection schemes. In this paper, we model SCAs as a communication channel and aim to suppress information leakage by minimizing the mutual information between the secret information and side-channel observations, subject to a power constraint on the artificial noise. We propose an optimal artificial noise injection method to minimize the mutual information in systems with Gaussian inputs. Specifically, we formulate two convex optimization problems: 1) minimizing the total mutual information, and 2) minimizing the maximum mutual information across observations. Numerical results show that the proposed methods significantly reduce both total and maximum mutual information compared to conventional techniques, confirming their effectiveness for resource-constrained, security-critical systems.

I. INTRODUCTION

Side-channel attacks (SCAs) pose a serious threat to cryptographic security by targeting confidential information such as secret keys or messages through physical leakages, including power consumption, timing variations, and electromagnetic emissions [1]–[3]. These attacks exploit variations in physical characteristics during secret-dependent operations and include power analysis attacks [4], timing attacks [5], and electromagnetic attacks [6]. Among these, power analysis attacks are especially effective, as they rely on monitoring a device's power consumption during cryptographic computations. By analyzing the resulting leakage traces, attackers aim to infer secret information [5], [7].

To defend against SCAs, several countermeasures have been proposed, including artificial noise injection [8], masking [9], and hiding techniques [10]. Among these, artificial noise injection aims to reduce the signal-to-noise ratio (SNR) of the observed leakage by adding external noise [3], [8], [11]. Although effective in principle, this approach is often inefficient in practice, as it typically requires noise power several times greater than the actual power consumption of encryption algorithms. This leads to substantial overhead, making the method impractical for resource-constrained devices.

Such constraints are particularly pronounced in Internet of Things (IoT) devices, which are susceptible to SCAs due to their limited power and computational resources [12], [13]. While high-resource systems can implement artificial noise injection effectively, IoT devices often lack the capacity to support these high-overhead techniques. Consequently, they remain exposed to power analysis attacks that exploit distinct power signatures during cryptographic operations.

We propose an artificial noise injection method that addresses the limitations of existing approaches by using mutual information [14] as a measure of side-channel information leakage. Our approach formulates an optimization problem that minimizes the mutual information under a power constraint on the artificial noise, with particular focus on applicability to resource-constrained devices.

Accurately quantifying information leakage is critical for evaluating the effectiveness of SCA countermeasures. While maximal leakage [15] is a well-known metric, the mutual information is more appropriate in the context of power analysis attacks, as it accommodates continuous random variables typically observed in side-channel measurements. In contrast, the maximal leakage diverges under these conditions.

We formulate two optimization problems: 1) minimizing the total mutual information across all leakage points (averagecase scenario), and 2) minimizing the maximum mutual information among them (worst-case scenario). Our method optimally allocates artificial noise across leakage points for both cases within a unified framework, yielding analytical solutions that can be interpreted as *dual* water-filling techniques. Compared to conventional uniform noise allocation, the proposed method significantly reduces the required artificial noise power while achieving the same level of leakage suppression.

The remainder of this paper is organized as follows. Section II provides background on the system model and information leakage metrics used in SCA problems. Section III formulates the artificial noise allocation problems and presents the corresponding optimal solutions. Section IV provides the numerical results that validate the proposed method. Section V concludes the paper.

II. BACKGROUNDS

A. Side-Channel Model

Suppose that the random variable U denotes the secret information, such as a secret key or message, and the observed sidechannel output signal, e.g., power consumption, is represented by the random vector $Y^m = \{Y_i\}_{i=1}^m = (Y_1, \ldots, Y_m) \in \mathbb{R}^m$. We introduce an intermediate random vector $X^m \in \mathbb{R}^m$, defined as $X^m = f(U)$, where $f(\cdot)$ is a (possibly stochastic) function that models the side-channel of the secret information. Let Z^m and N^m denote the physical and artificial noise vectors, respectively. Both noise vectors are modeled as Gaussian distributions; specifically, $Z_i \sim \mathcal{N}(0, Z_i)$ and $N_i \sim \mathcal{N}(0, N_i)$ for $i = 1, \ldots, m$. The side channel observation model is then given by

$$Y^{m} = f(U) + N^{m} + Z^{m} = X^{m} + N^{m} + Z^{m}, \quad (1)$$

where the power of each intermediate component is denoted by $\mathbb{E}[X_i^2] = \mathsf{P}_i$.

B. Information Leakage Metrics for SCA

In this subsection, we introduce representative information leakage metrics and discuss related works on SCAs.

1) Mutual information: The mutual information quantifies the amount of information shared between two random variables. In the context of SCAs, the mutual information measures how much information about the secret variable Ucan be inferred from the side-channel output Y^m . It is defined as

$$I(U;Y^m) = \sum_{y^m \in \mathcal{Y}^m} \sum_{u \in \mathcal{U}} P(u, y^m) \log\left(\frac{P(u, y^m)}{P(u)P(y^m)}\right).$$
(2)

This metric is widely adopted in both discrete and continuous settings for quantifying information leakage, due to its solid theoretical foundation and operational relevance in SCA evaluation [3], [16]. Jin *et al.* [3] adopted the Gaussian channel capacity–i.e., mutual information under a Gaussian input assumption–as the metric for evaluating information leakage in power analysis attacks. They formulated an optimization problem to control this capacity by injecting artificial noise that is *uniformly allocated* across a subset of leakage points.

2) Maximal leakage: The maximal leakage quantifies the maximum gain in an adversary's ability to guess the secret variable U after observing the side channel output Y^m , compared to making a blind guess [15]. The maximal leakage is defined as

$$L(X^m \to Y^m) = \sup_{U \to X^m \to Y^m \to \hat{U}} \log\left(\frac{\Pr(U = \hat{U})}{\max_u P(u)}\right), \quad (3)$$

where \hat{U} denotes the estimate of U inferred from Y^m . While the maximal leakage provides a worst-case guarantee, it is not suitable for continuous variables, as it diverges in such cases [15]. In [17], the authors minimize the maximal leakage by injecting artificial delay in discrete time. 3) Sibson mutual information: The Sibson mutual information is a generalization of mutual information, parameterized by a tunable order $\alpha > 0$ [18], [19]. It is defined as

$$I_{\alpha}(U;Y^m) = \inf_{P_{Y^m}} D_{\alpha}(P_{UY^m} || P_U \times P_{Y^m}), \qquad (4)$$

where $D_{\alpha}(P||Q)$ denotes the Rényi divergence of order α . As $\alpha \to 1$, $I_{\alpha}(U; Y^m)$ converges to the mutual information. As $\alpha \to \infty$, it converges to the maximal leakage [15].

III. OPTIMAL ARTIFICIAL NOISE FOR SIDE CHANNEL ANALYSIS RESISTANCE

An attacker attempts to infer the secret variable U from the side-channel observation Y^m , and our objective is to increase the probability of inference error $P_e = \Pr(U \neq \hat{U})$. According to Fano's inequality [14], the probability of inference error satisfies

$$1 + P_e \log |\mathcal{U}| \ge H(U|Y^m),\tag{5}$$

where \mathcal{U} denotes the alphabet of U. This inequality implies that a larger conditional entropy $H(U|Y^m)$ leads to a higher lower bound on P_e . Since the mutual information is defined as $I(U;Y^m) = H(U) - H(U|Y^m)$, minimizing $I(U;Y^m)$ is equivalent to maximizing $H(U|Y^m)$, which in turn increases the lower bound on the probability of inference error. Hence, we aim to minimize the mutual information $I(U;Y^m)$.

We assume that the relationship between the secret information U and the intermediate vector X^m is immutable, and the system designer has control over how Y^m is generated from X^m , as in [17]. Hence, instead of directly minimizing the mutual information $I(U;Y^m)$, we aim to minimize its upper bound, which simplifies the optimization problem. Since $U \to X^m \to Y^m$ forms a Markov chain, the following inequality holds by the data processing inequality:

$$I(U; Y^m) \le I(X^m; Y^m) \le \sum_{i=1}^m I(X_i; Y_i).$$
 (6)

Moreover, this data processing inequality also holds for the Sibson mutual information [19], making it applicable in more general settings involving α -parametrized leakage measures.

We aim to minimize information leakage by injecting artificial noise under a fixed artificial-noise power budget. As information-leakage metrics, we adopt the mutual information and the Sibson mutual information. However, the maximal leakage is not suitable in our setting, as both X and Y are continuous. Note that $L(X^m \to Y^m) \to \infty$ when X and Y are continuous [15, Example 10].

We can then formulate the following optimization problem:

$$\begin{array}{l} \underset{\{\mathsf{N}_i\}_{i=1}^m}{\min } \sum_{i=1}^m I(X_i; Y_i) \\ \text{subject to } \sum_{i=1}^m \mathsf{N}_i \le \mathsf{N}_0, \quad \mathsf{N}_i \ge 0. \end{array}$$
(7)

where N₀ denotes the total artificial noise power constraint. Note that from (1) each mutual information term $I(X_i; Y_i)$ is a function of N_i for i = 1, ..., m.

A. Minimizing Total Mutual Information

Similarly to [3], we adopt a sub-channel perspective, where each leakage point in the side channel is modeled as a distinct sub-channel. The amount of leakage is quantified by the subchannel powers, P_i , i = 1, ..., m, and is assumed to be known. In addition, we assume that the variances Z_i of the physical noise are known. We focus on the case where each input X_i follows a Gaussian distribution and aim to minimize an upper bound on the mutual information.

$$I(X_i; Y_i) \le \frac{1}{2} \log \left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i} \right),\tag{8}$$

which corresponds to the channel capacity of an additive Gaussian noise channel. Substituting this upper bound into the original objective (7), the optimization problem becomes:

$$\begin{array}{l} \underset{\{\mathsf{N}_i\}_{i=1}^m}{\text{minimize}} \quad \sum_{i=1}^m \log\left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i}\right) \quad (9) \\ \text{subject to} \quad \sum_{i=1}^m \mathsf{N}_i \le \mathsf{N}_0, \quad \mathsf{N}_i \ge 0. \end{array}$$

Lemma 1: The optimization problem in (9) is convex with respect to $\{N_i\}_{i=1}^m$.

Proof: For $P_i \ge 0$, the second derivative of the objective function is given by

$$\frac{\partial^2}{\partial \mathsf{N}_i^2} \sum_{i=1}^m \log\left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i}\right)$$
$$= \frac{1}{(\mathsf{N}_i + \mathsf{Z}_i)^2} - \frac{1}{(\mathsf{N}_i + \mathsf{Z}_i + \mathsf{P}_i)^2} \ge 0, \tag{10}$$

which confirms the convexity of the objective function.

The optimal solution can be derived by applying the Karush-Kuhn-Tucker (KKT) conditions.

Theorem 1: The optimal noise allocation $\{N_i^*\}_{i=1}^m$ of (9) is given by

$$\mathsf{N}_{i}^{*} = \begin{cases} 0, & \nu \geq \frac{1}{\mathsf{Z}_{i}} - \frac{1}{\mathsf{Z}_{i} + \mathsf{P}_{i}}, \\ \frac{-(2\mathsf{Z}_{i} + \mathsf{P}_{i}) + \sqrt{\mathsf{P}_{i}^{2} + \frac{4\mathsf{P}_{i}}{\nu}}}{2}, & \text{otherwise}, \end{cases}$$
(11)

where ν is the dual variable. Also, for $N_i^* > 0$, the optimal N_{i}^{*} satisfies the following condition:

$$\frac{1}{\mathsf{N}_{i}^{*} + \mathsf{Z}_{i}} - \frac{1}{\mathsf{N}_{i}^{*} + \mathsf{Z}_{i} + \mathsf{P}_{i}} = \nu \tag{12}$$

Proof: We consider only the case where $P_i > 0$, as the mutual information becomes zero when $P_i = 0$, indicating the absence of information leakage. In such cases, allocating artificial noise power is unnecessary, and the optimal noise allocation is naturally zero. Therefore, we focus on components with $P_i > 0$. We define the Lagrangian $\mathcal{L}_1(N_i, \lambda_i, \nu)$ associated with (9) as follows:

$$\mathcal{L}_{1}(\mathsf{N}_{i},\lambda_{i},\nu) = \sum_{i=1}^{m} \log\left(1 + \frac{\mathsf{P}_{i}}{\mathsf{N}_{i} + \mathsf{Z}_{i}}\right) - \sum_{i=1}^{m} \lambda_{i}\mathsf{N}_{i} + \nu\left(\sum_{i=1}^{m}\mathsf{N}_{i} - \mathsf{N}_{0}\right), \quad (13)$$

where λ_i and ν are the dual variables. We have the following KKT conditions:

$$\lambda_i \ge 0, \quad \nu \ge 0, \quad \frac{\partial \mathcal{L}_1}{\partial \mathsf{N}_i} = 0,$$
 (14)

$$\lambda_i \cdot \mathsf{N}_i = 0, \quad \nu\left(\sum_{i=1}^m \mathsf{N}_i - \mathsf{N}_0\right) = 0. \tag{15}$$

From the KKT conditions, we obtain

$$\lambda_i = \frac{1}{N_i + Z_i + P_i} - \frac{1}{N_i + Z_i} + \nu \ge 0.$$
 (16)

If $\nu = 0$, then $\lambda_i < 0$ for $\mathsf{P}_i > 0$, which violates the KKT conditions $\lambda_i \ge 0$. Hence, we should have $\nu > 0$, and by (15), $\sum_{i=1}^{m} N_i = N_0$. The condition $\lambda_i \cdot N_i = 0$ of (15) leads to

$$N_i \left(\frac{1}{N_i + Z_i + P_i} - \frac{1}{N_i + Z_i} + \nu \right) = 0.$$
 (17)

We now consider three cases based on the value of ν relative to the threshold $\frac{1}{Z_i} - \frac{1}{Z_i + P_i}$:

- If $\nu > \frac{1}{Z_i} \frac{1}{Z_i + P_i}$, then we have $\lambda_i > 0$ by (16), and thus $N_i = 0$ by (15).
- If $\nu = \frac{1}{Z_i} \frac{1}{Z_i + P_i}$, then (17) holds for $N_i = 0$. If $\nu < \frac{1}{Z_i} \frac{1}{Z_i + P_i}$, then setting $N_i = 0$ leads to $\lambda_i < 0$ in (16), which violates (14). Hence, we should have $N_i > 0$ and $\lambda_i = 0$, which leads to (12). Solving (12) yields the optimal solution in (11).

By combining these three cases, we obtain the optimal solution given in (11).

Our optimization problem in (9) can be viewed as a *dual* to the classical water-filling problem [14], which maximizes the total capacity of the parallel Gaussian channels by allocating transmit power $\{\mathsf{P}_i\}_{i=1}^m$ across channels. In the water-filling problem, artificial noise is not considered; the optimization variables are $\{\mathsf{P}_i\}_{i=1}^m$ for a given set of noise power $\{\mathsf{Z}_i\}_{i=1}^m$. For the selected channels, the optimal power allocation satisfies the condition $P_i^* + Z_i = \nu$, where ν represents the water-level. In contrast, our objective is to minimize the total capacity of parallel Gaussian channels by allocating artificial noise $\{N_i\}_{i=1}^m$. For the selected points (i.e., those with nonzero artificial noise allocation), the optimal noise allocation N_i^* satisfies the condition in (12), which serves as the dual counterpart to the optimal P_i^* in the water-filling solution.

The optimization problem can be extended to the setting of Sibson mutual information. According to [19], for a Gaussian channel with a Gaussian input, the Sibson mutual information for $\alpha > 0$ is given by

$$I_{\alpha}(X_i; Y_i) = \frac{1}{2} \log \left(1 + \alpha \cdot \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i} \right).$$
(18)

Then, the corresponding optimization problem is as follows:

$$\begin{array}{l} \underset{\{\mathsf{N}_i\}_{i=1}^m}{\text{minimize}} & \sum_{i=1}^m \log\left(1 + \frac{\alpha \mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i}\right) \\ \text{subject to} & \sum_{i=1}^m \mathsf{N}_i \le \mathsf{N}_0, \quad \mathsf{N}_i \ge 0. \end{array}$$

$$(19)$$

As in Lemma 1, this optimization problem can be shown to be convex. Moreover, we can derive the optimal solution as in Theorem 1.

Corollary 1: The optimal noise allocation $\{N_i^*\}_{i=1}^m$ of (19) is given by:

$$\mathsf{N}_{i}^{*} = \begin{cases} 0, & \nu \geq \frac{1}{\mathsf{Z}_{i}} - \frac{1}{\mathsf{Z}_{i} + \alpha \mathsf{P}_{i}}, \\ \frac{-(2\mathsf{Z}_{i} + \alpha \mathsf{P}_{i}) + \sqrt{(\alpha \mathsf{P}_{i})^{2} + \frac{4\alpha \mathsf{P}_{i}}{\nu}}}{2}, & \text{otherwise}, \end{cases}$$
(20)

where ν is the dual variable. Also, for $N_i^* > 0$, the noise allocation $\{N_i^*\}_{i=1}^m$ satisfies the following condition:

$$\frac{1}{\mathsf{N}_{i}^{*} + \mathsf{Z}_{i}} - \frac{1}{\mathsf{N}_{i}^{*} + \mathsf{Z}_{i} + \alpha \mathsf{P}_{i}} = \nu.$$
(21)

Our framework is flexible and adaptable to generalized information leakage measures, where α serves as a tunable parameter that modulates the sensitivity to signal power.

Remark 1: As $\alpha \rightarrow \infty$, the Sibson mutual information converges to the maximal leakage [15]. However, for Gaussian inputs and Gaussian channel noise, the maximal leakage diverges, since (18) goes to infinity as $\alpha \to \infty$. Hence, the maximal leakage is not suitable for scenarios involving continuous side-channel inputs and outputs.

B. Minimizing Maximum Mutual Information

In this subsection, we focus on the pointwise maximum mutual information, as such points may indicate severe information leakage. This approach is motivated by prior work [16], where the pointwise maximum mutual information was used to identify critical leakage points. To address this, we aim to minimize the pointwise maximum mutual information, i.e., $\max_{i \in \{1,...,m\}} I(X_i; Y_i)$. The corresponding optimization problem is formulated as follows:

$$\begin{array}{l} \underset{\{\mathsf{N}_i\}_{i=1}^m}{\text{minimize}} & \underset{i \in \{1, \dots, m\}}{\max} \log \left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i} \right) \quad (22) \\ \text{subject to} & \sum_{i=1}^m \mathsf{N}_i \leq \mathsf{N}_0, \quad \mathsf{N}_i \geq 0. \end{array}$$

Since this problem is convex, we can obtain the optimal noise allocation.

Theorem 2: The optimal noise allocation $\{N_i^*\}_{i=1}^m$ of (22) is given by

$$\mathsf{N}_{i}^{*} = \begin{cases} 0, & \kappa \geq \frac{\mathsf{P}_{i}}{\mathsf{Z}_{i}}, \\ \frac{-(2\mathsf{Z}_{i}+\mathsf{P}_{i})+\sqrt{\mathsf{P}_{i}^{2}+\frac{4\mathsf{P}_{i}^{2}(1+\kappa)}{\kappa^{2}}}}{2}, & \text{otherwise}, \end{cases}$$
(23)

where κ is given by

$$\kappa = \frac{\mathsf{P}_i \nu + \sqrt{(\mathsf{P}_i \nu)^2 + 4\mathsf{P}_i \nu \eta_i}}{2\eta_i}.$$
 (24)

Here, ν and η_i are the dual variables. For $N_i^* > 0$, the optimal N_i^* satisfies the following condition:

$$\eta_i \left(\frac{1}{\mathsf{N}_i^* + \mathsf{Z}_i} - \frac{1}{\mathsf{N}_i^* + \mathsf{Z}_i + \mathsf{P}_i} \right) = \nu.$$
 (25)

Proof: The problem (22) can be reformulated as the following equivalent problem:

$$\begin{array}{ll} \underset{\{\mathsf{N}_i\}_{i=1}^m}{\underset{i=1}{\overset{m}{\longrightarrow}}} & \xi & (26) \\ \text{subject to} & \sum_{i=1}^m \mathsf{N}_i \le \mathsf{N}_0, \quad \mathsf{N}_i \ge 0, \\ & \log\left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i}\right) \le \xi. \end{array}$$

We consider only the case where $P_i > 0$, for the same reason as in the proof of Theorem 1. We define the Lagrangian $\mathcal{L}_2(N_i, \lambda_i, \nu)$ associated with (26) as follows:

$$\mathcal{L}_{2}(\mathsf{N}_{i},\xi,\lambda_{i},\nu,\eta_{i}) = \xi - \sum_{i=1}^{m} \lambda_{i}\mathsf{N}_{i} + \nu \left(\sum_{i=1}^{m} \mathsf{N}_{i} - \mathsf{N}_{0}\right) + \sum_{i=1}^{m} \eta_{i} \left(\log\left(1 + \frac{\mathsf{P}_{i}}{\mathsf{N}_{i} + \mathsf{Z}_{i}}\right) - \xi\right), \quad (27)$$

where ν, λ_i , and η_i are the dual variables. We have the following KKT conditions:

$$\lambda_i \ge 0, \quad \nu \ge 0, \quad \eta_i \ge 0, \tag{28}$$

$$\frac{\partial L_2}{\partial \mathsf{N}_i} = 0, \quad \frac{\partial L_2}{\partial \xi} = 0,$$
 (29)

$$\mathbf{N}_i \cdot \mathbf{N}_i = 0, \quad \nu\left(\sum_{i=1}^m \mathbf{N}_i - \mathbf{N}_0\right) = 0,$$
 (30)

$$\eta_i \left(\log \left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i} \right) - \xi \right) = 0.$$
(31)

From (28) and (29), we obtain

$$\lambda_{i} = \frac{\eta_{i}}{\mathsf{N}_{i} + \mathsf{Z}_{i} + \mathsf{P}_{i}} - \frac{\eta_{i}}{\mathsf{N}_{i} + \mathsf{Z}_{i}} + \nu \ge 0, \quad \sum_{i=1}^{m} \eta_{i} = 1.$$
(32)

If $\nu = 0$, then for $\mathsf{P}_i > 0$, either $\lambda_i < 0$ or $\eta_i = 0$ should hold, both of which violate the KKT conditions $\lambda_i \ge 0$ and $\sum_{i=1}^{m} \eta_i = 1$, respectively. Hence, we should have $\nu > 0$, and by (30), $\sum_{i=1}^{m} N_i = N_0$. The condition $\lambda_i \cdot N_i = 0$ of (30) leads to

$$\mathsf{N}_i \left(\frac{\eta_i}{\mathsf{N}_i + \mathsf{Z}_i + \mathsf{P}_i} - \frac{\eta_i}{\mathsf{N}_i + \mathsf{Z}_i} + \nu \right) = 0.$$
(33)

We define the positive constant $\kappa = e^{\xi} - 1$, and consider three cases based on its value relative to the threshold $\frac{P_i}{Z_i}$:

- If κ > P_i/Z_i, then η_i = 0 by (31), and thus N_i = 0 by (33).
 If κ = P_i/Z_i, then (31) holds for N_i = 0.
- If $\kappa < \frac{\mathsf{P}_i}{\mathsf{Z}_i}$, then $\mathsf{N}_i = 0$ leads to $\log\left(1 + \frac{\mathsf{P}_i}{\mathsf{N}_i + \mathsf{Z}_i}\right) > \xi$, which violates $\log \left(1 + \frac{P_i}{N_i + Z_i}\right) \le \xi$ of (26). Hence, we should have $N_i > 0$ and $\lambda_i = 0$, which leads to (25), and thus $\eta_i > 0$. It implies $\log \left(1 + \frac{P_i}{N_i + Z_i}\right) = \xi$ by (31). Then, we obtain

$$\mathsf{N}_i = \frac{\mathsf{P}_i}{\kappa} - \mathsf{Z}_i. \tag{34}$$



Fig. 1. Comparison of average (total) mutual information under optimal artificial noise allocation and uniform artificial noise allocation. (a) represents a low SNR scenario with $P_i \sim \mathcal{N}(1, 0.5^2)$, $Z_i = 1000$. (b) represents a high SNR scenario with $P_i \sim \mathcal{N}(1, 0.5^2)$, $Z_i = 100$.

Substituting (34) into (25), we obtain

$$\eta_i = \mathsf{P}_i \cdot \frac{\nu(1+\kappa)}{\kappa^2}.$$
(35)

Finally, Solving (25) with (35) yields (23).

By combining these three cases, we obtain the optimal solution given in (23).

It is worth mentioning that (22) addresses the worst-case scenario, while (9) corresponds to the average-case scenario.

IV. NUMERICAL RESULTS

In this section, we evaluate the effectiveness of the proposed optimal artificial noise allocation. Specifically, we compare three approaches: (1) no artificial noise allocation, (2) uniform artificial noise allocation, and (3) optimized artificial noise allocation. According to [11], the average current consumption during advanced encryption standard (AES) operations is approximately 1 mA, while effective artificial noise injection requires about 17 mA. Given that power scales with the square of current under constant resistance, this corresponds to a power ratio of approximately 1 : 300. Based on this, the average power consumption is normalized to 1, and the



Fig. 2. Comparison of average (total) mutual information under optimal artificial noise allocation and uniform artificial noise allocation (a) represents a low SNR scenario with $P_i \sim \mathcal{U}[0,2]$, $Z_i = 1000$. (b) represents a high SNR scenario with $P_i \sim \mathcal{U}[0,2]$, $Z_i = 100$.

average power of artificial noise is varied from 1 to 300. To evaluate the impact of artificial noise, we varied the total noise power $N_0 = \sum_{i=1}^{m} N_i$ from 0 to $300 \times m$, with m = 100. To capture diverse scenarios, the input power P_i is sampled from Gaussian and uniform distributions. In the Gaussian case, negative values are set to zero to ensure non-negative power values. Recognizing that physical noise levels may vary across devices, our experimental design includes both low and high SNR regimes to ensure comprehensive evaluation.

Fig. 1 and Fig. 2 show that the proposed optimal noise allocation method consistently outperforms uniform allocation in reducing the average (total) mutual information, with P_i sampled from Gaussian and uniform distributions, respectively. Specifically, to achieve a 50 % reduction in the average mutual information, the proposed method requires 19.14 % and 19.13 % less artificial noise power than the uniform allocation in Fig. 1(a) and Fig. 1(b), respectively. Similarly, the proposed method reduces artificial noise power by 22.29 % and 22.21 % in Fig. 2(a) and Fig. 2(b), respectively. These results clearly indicate that the proposed method significantly improves noise power efficiency compared to uniform noise allocation, which is particularly critical for resource-constrained devices.



Fig. 3. Comparison of the maximum mutual information under optimal artificial noise allocation and uniform artificial noise allocation (a) represents $P_i \sim \mathcal{N}(1, 0.5^2)$, $Z_i = 1000$. (b) represents $P_i \sim \mathcal{U}[0, 2]$, $Z_i = 1000$.

In the case of maximum mutual information minimization, the advantages of the proposed method become even more pronounced. To reduce the pointwise maximum mutual information to half its original value, the proposed method reduces the required artificial noise power by 87.84% and 73.53%, respectively, as shown in Fig. 3(a) and Fig. 3(b). This substantial reduction shows the effectiveness of the proposed method in mitigating peak information leakage.

V. CONCLUSION

We presented a principled information-theoretic framework for protecting cryptographic systems from power analysis attacks through optimal artificial noise injection. Our approach aims to minimize both the total mutual information and the maximum mutual information. We derived optimal noise allocation solutions, which can be interpreted as the dual problem of the classical water-filling. Numerical results show that our proposed method reduces the mutual information more effectively than conventional uniform allocation.

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