Merchants of Vulnerabilities: How Bug Bounty Programs

Benefit Software Vendors

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Abstract: Software vulnerabilities enable exploitation by malicious hackers, compromising systems and data security. This paper examines bug bounty programs (BBPs) that incentivize ethical hackers to discover and responsibly disclose vulnerabilities to software vendors. Using game-theoretic models, we capture the strategic interactions between software vendors, ethical hackers, and malicious hackers. First, our analysis shows that software vendors can increase expected profits by participating in BBPs, explaining their growing adoption and the success of BBP platforms. Second, we find that vendors with BBPs will release software earlier, albeit with more potential vulnerabilities, as BBPs enable coordinated vulnerability disclosure and mitigation. Third, the optimal number of ethical hackers to invite to a BBP depends solely on the expected number of malicious hackers seeking exploitation. This optimal number of ethical hackers is lower than but increases with the expected malicious hacker count. Finally, higher bounties incentivize ethical hackers to exert more effort, thereby increasing the probability that they will discover severe vulnerabilities first while reducing the success probability of malicious hackers. These findings highlight BBPs' potential benefits for vendors beyond profitability. Earlier software releases are enabled by managing risks through coordinated disclosure. As cybersecurity threats evolve, BBP adoption will likely gain momentum, providing vendors with a valuable tool for enhancing security posture and stakeholder trust. Moreover, BBPs envelop vulnerability identification and disclosure into new market relationships and transactions, impacting software vendors' incentives regarding product security choices like release timing.

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1 Introduction

Marc Andreessen famously stated in 2011 that 'software is eating the world,' transforming not only industries traditionally associated with technology but also those primarily existing in the physical realm (Andreessen 2011). However, this pervasive software influence has a downside: vulnerabilities. The National Institute of Standards and Technologies (NIST) describes software vulnerability as a "security flaw, glitch, or weakness found in software code that could be exploited by an attacker (threat source)" (NIST CSRC 2020). Unlike physical products, software vulnerabilities can be exploited remotely, posing a unique challenge. For instance, Sam Curry and other ethical hackers discovered that by exploiting a vulnerability in Sirius XM, a satellite radio service widely used in vehicles, they could gain remote access to the vehicle by using the unique vehicle identification number (VIN). The VIN is often publicly available (e.g., on websites of car dealers) or can be read on the windshield of some vehicles. Due to the software vulnerability, hackers could remotely start, unlock, locate, flash the lights, and honk the horn on the car (Ropek 2022). Sirius XM eventually fixed the vulnerability based on the report submitted by these ethical hackers. However, while this vulnerability existed on the vehicles, a malicious car-hacking-as-a-service platform could have teamed up with thieves to allow them to steal valuables by remotely unlocking cars!

The scourge of software vulnerabilities (SVs), or bugs, ¹ has long plagued software products. Yet a consensus on how to disclose these SVs effectively and responsibly remains elusive. The problem stems from the negative externality imposed by SV disclosure: in addition to legitimate recipients, malicious hackers also learn about the SV from the disclosure and can begin exploiting it. Thus, on average, product vendors prefer the non-disclosure of specific SVs to avoid this negative externality and reduce the cost of quickly developing and distributing a fix for the SV through patching. The vendor can control the patching process completely if only the vendor is aware of the SV. However, when a third party, such as a product user, discovers and reports an SV, the third-party expects the vendor to fix the SV through a patch immediately and inform all users. On both these counts, the vendors' incentives are misaligned — they prefer to delay patches to reduce costs and control disclosure to avoid the externality. Thus, vendors may obfuscate or ignore third-party SV reports, delay patches by not allocating resources on a priority basis, or fix the vulnerability in the new version without releasing a timely patch. These vendor behaviors are sub-optimal for users and have led many users to disclose SVs publicly to force vendors to fix the SVs on a priority basis. However, such uncoordinated public disclosures have generated acrimony, and vendors have demanded that any user who discovers an SV should practice "responsible disclosure." That is, the discovering user should first make a private disclosure to the software vendor and provide a reasonable time for the vendor to fix the SV before the user does a public disclosure.

¹ Although bugs and software vulnerabilities are slightly different, we use the terms interchangeably.

One approach for dealing with SVs that has become increasingly popular is the use of bug bounty programs (BBP) that not only help vendors discover the SVs but also have better control over the SV disclosures. The BBPs allow vendors to crowd-source SV discovery by systematically engaging external security experts (or ethical hackers) to discover and report bugs in the software in exchange for monetary rewards. Besides receiving monetary rewards, these security experts also enhance their professional reputation when they successfully participate in a BBP. The aforementioned Sirius XM bug discovery is an example of a bug being discovered and reported by an external security expert, although not as a part of a BBP. BBPs make this bug discovery process more systematic by setting rules of engagement, providing immunity from prosecution for activities conducted within the established rules,² and incentivizing security experts to proactively engage with the software to discover and report bugs to the vendor. There are several advantages of BBPs for the vendors but two worth noting here—BBPs allow vendors to discover SVs that may have escaped notice of their development and testing teams and exercise greater control over SV disclosures. In some cases, the vendor's BBP may have an explicit rule that prohibits disclosure as a condition for participation by the security expert, thus allowing the vendor to achieve its ideal of non-disclosure in these cases. Although bug bounty programs have existed since at least 1995, when the pioneering browser vendor Netscape established a bug bounty program to control uncoordinated disclosure of bugs, the widespread establishment of bug bounty programs by many US and global software vendors is a more recent

² Outside of bug bounty programs, even good Samaritans may face legal threats or prosecution for identifying and disclosing software vulnerabilities (Treisman 2021).

phenomenon. The BBPs are "one prominent instance of coordinated [software] vulnerability disclosure—a set of approaches that have the potential to routinize the reporting and disclosure of flaws, improve security, and buffer the risks of legal reprisal" (Ryan Ellis and Yuan Stevens 2022) and are now an important part of the cybersecurity strategy of many software vendors.

Another important aspect of BBPs is their impact on fostering the ethical hacking community. For many ethical hackers, bug bounty programs offer new incentives, career prospects, and a legitimate avenue to leverage their skills. Rather than facing potential legal repercussions for disclosing flaws, these programs enable ethical hackers to gain recognition and monetary rewards when they responsibly report vulnerabilities to vendors. However, BBPs represent more than just a mechanism for matching hackers with companies seeking to identify bugs. Through structured programs with defined rules of engagement and incentives, BBPs have introduced a market-based approach to vulnerability management. This shift extends beyond simply paying bounties – it has enveloped the entire identification and disclosure cycle within a framework of market relationships and transactions between vendors and the ethical hacker community (Ryan Ellis and Yuan Stevens 2022).

Despite their popularity, it is not clear whether BBPs improve software quality. Do they encourage vendors to invest more resources in improving quality? How do vendors decide how much bounty they should set to encourage ethical hackers to report the bugs? Finally, do BBPs impact vendors' decision on when to release the product? Specifically, do BBPs enable vendors to release the product early to capture

market share? In this study, we introduce a game-theoretic framework to analyze the strategic interactions between software vendors, ethical hackers, and malicious hackers. We explore how BBPs influence the incentives for ethical and malicious hackers to discover vulnerabilities and the potential externalities arising from their actions. Additionally, we investigate the effects of BBPs on software vendors' security incentives, with a focus on how these programs impact the timing of software releases.

The first key finding from our analysis is that under plausible assumptions, a software vendor will increase its expected profit by participating in a BBP. This finding explains BBP's growing popularity among software vendors and the success of bug bounty program platforms such as HackerOne. The second key finding is that software vendors with BBP will release their software earlier than those without BBP. As testing for vulnerabilities and fixing them takes time, an earlier release implies that software vendors with BBP will release software with comparatively more bugs. While this may increase the risk for software users, the vendor can mitigate the risk of releasing buggier software, as ethical hackers in the BBP would find some of these vulnerabilities and would be contractually obligated to coordinate disclosure with the software vendor. The third key finding is that if the vendor can choose the number of ethical hackers it wants to invite into its BBP, the optimal number of ethical hackers only depends on the expected number of malicious hackers likely to look for vulnerabilities for exploitation. In particular, the number of optimal ethical hackers is less than the expected number of malicious hackers and increases as the expected number of malicious hackers increases.

Finally, higher bounties incentivize ethical hackers to exert more effort, thereby increasing the probability that they will discover severe vulnerabilities first while reducing the success probability of malicious hackers.

2 Background and Current Literature

Software vendors resort to a variety of approaches to mitigate the risk of releasing software with major vulnerabilities. Most well-managed software vendors perform rigorous pre-release software testing, including manual and automated testing methods, along with code reviews that scrutinize source code for potential vulnerabilities. However, the maxim by Turing award-winning computer scientist Edsger Dijkstra, "program testing can be used to show the presence of bugs, but never to show their absence!" (Dijkstra 2022), also applies to security vulnerabilities. Thus, it is impossible to prove that a commercial software product has no vulnerabilities (Thompson 1984), although on average, vulnerabilities can be reduced with testing.

Software code reviews and testing are costly for the vendors as they require resource allocation, such as investing in testing and review tools and hiring and retaining code reviewers and testers. Code reviews and testing also delay the release of the product that incorporates the software, with implications such as lost revenue and lost market share. Furthermore, any amount of in-house testing, no matter how well-planned and well-executed, will rarely subject software to all the real-world use cases from legitimate users or abuse cases from malicious adversaries. While testing techniques that mimic methods of malicious adversaries,

such as fuzzing, "a process of repeatedly running a program with generated inputs that may be syntactically or semantically malformed" (Manès et al. 2019), have been adopted by software vendors, no manual or automated process can anticipate and test all possibilities.

Hence, software vendors are faced with the decision before every software launch: should they release the software at a particular time point with a higher risk of a severe vulnerability being discovered after software release, or should they delay the software release and test more to lessen the risk of post-release discovery of a vulnerability. Early release can result in higher revenue and market share for the vendor but increases the risk of a post-release discovery of a vulnerability. On the one hand, more testing can reduce the risk of releasing software with a vulnerability but on the other hand, it may lead to a loss of revenue and market share. At the time of release, vendors know that software may still carry a non-zero likelihood of containing severe bugs. However, they decide to ship the product only when the risk posed by these software vulnerabilities has been mitigated to a tolerable level. Vendors rely on post-release strategies such as incident response plans, cybersecurity insurance, patch management, and bug bounty programs to manage residual vulnerabilities. Incident response plans and cybersecurity insurance primarily attempt to contain vendors' losses after a bug has been exploited. In contrast, bug bounty programs and patch management primarily aim to identify and fix bugs before exploitation. A patch is a piece of code that can be applied to already released software to correct, improve, or update it. The process of applying a patch is known as patching, whereas the overall management, including assessment,

testing, and deployment of patches, is called patch management. While patches may be used to add new functionality, they primarily enable companies to fix bugs after a product has been shipped, a practice that, surprisingly, may be socially optimal despite allowing the release of buggier products (Arora, Caulkins, and Telang 2006). As such, bugs are often discovered throughout a product's lifetime, prompting vendors to issue patches in response. The speed at which these patches are released varies among software vendors, with faster patch times notably associated with greater market competition, the public disclosure of vulnerabilities, and the larger scale of the vendor (Arora, Forman, et al. 2010; Arora, Krishnan, et al. 2010).

2.1 Bug Bounty Programs

Patching, a process that can only commence once a bug has been identified, addresses known bugs. Bug discoveries in the post-release period can occur through various channels: vendors' internal testing, user-reported bugs, vulnerability disclosures by entities like the Computer Emergency Readiness Team Coordination Center (CERT/CC), and bug bounty programs. Bug bounty programs (BBPs) represent a distinct post-release strategy for identifying vulnerabilities.³ These programs provide an additional testing layer by incentivizing highly skilled security experts, not employed by the vendor, to find residual and often elusive software vulnerabilities. These programs allow companies to crowdsource bug findings by rewarding white hat hackers (WHH), also known as ethical hackers or independent security researchers (hitherto WHH) who report valid bugs. These WHHs may have

³ Although BBP may be used for pre-beta and beta testing, these are generally specialized and restricted programs (Malladi and Subramanian 2020).

specialized skills that may be too costly for a single vendor to find and hire but can be brought to bear periodically across multiple vendors through such BBPs. The "bounty" for discovering bugs incentivizes the WHH to spend their efforts finding and reporting residual vulnerabilities to the product vendor. The software vendors benefit from the BBP by leveraging the diverse talents, skills, and capabilities of WHHs, which are often unavailable within vendors' internal security teams.⁴ Moreover, bug bounty is only paid if a WHH reports an actionable bug and is paid only once to the first WHH who reports the bug to the product vendor. Thus, BBPs expend vendors' funds only if there is an actionable bug report⁵ whereas the vendors must invest in tools and in-house testing teams whether they find the bugs.

BBPs started with software vendors allowing ethical hackers to discover and disclose security vulnerabilities to software vendors in exchange for "bounties" or financial rewards. Large, well-known software vendors such as Google establish a "direct bug bounty" and are able to attract WHH to discover and report bugs (Ahmed, Deokar, and Lee 2021). However, smaller, lesser-known vendors may not attract WHH even if they establish BBP, as search costs would be too high for WHH. To facilitate search and matching, multi-sided platforms such as HackerOne, Bugcrowd, SynAck, Open Bug Bounty, etc., have entered the market to match software vendors with ethical hackers who hunt for bug bounties under program rules that are set by the vendor and affirmed by these bug bounty hunters. These BBP platforms not only facilitate search and matching but also provide other

⁴ Chris Nims, Oath Chief Information Security Officer

⁽https://www.youtube.com/watch?v=bsggw67EMcY)

⁵ Vendors have to spend resources to screen bug reports for validity. Hence, they do spend some resources in processing and rejecting invalid reports.

functions to both vendors and WHHs (e.g., independent evaluation of a bug report by a WHH). These platform-mediated BBPs have significantly enabled small and medium-sized vendors, who may not have otherwise successfully created a viable BBP independently.

As the adoption of BBPs has increased, researchers in computer science, economics, information systems, and other related fields have started to build a body of literature on BBPs. One strand of literature considers the costs and the implementation choices of BBPs.

Walshe and Simpson performed a descriptive empirical study of bug bounty programs and found them to be a "valuable complementary technique" for bug discovery (Walshe and Simpson 2020). Based on a simple back-of-the-envelope analysis, they assert that establishing a bug bounty program for a year costs less than hiring two additional software engineers. Feng et al. studied optimal timing for launching bug bounty programs relative to software release, comparing perpetual vs subscription licensing models. For perpetual licenses, simultaneous launch is best if failure cost or trust benefit is high, otherwise no bug bounty. For subscriptions, delayed bug bounty launch after software can be optimal when the failure cost is moderate and the trust benefit is significant (Feng et al. 2024). Zhou et al. analyze BBPs in the context of digital platforms (e.g., Google Play Store) and the third-party software sold through these platforms (T. Zhou, Ma, and Feng 2023). In their setup, platforms decide to launch BBPs and third-party software vendors decide to participate in BBPs. These decisions depend on the expected loss due to security

breach and the vendor's reliability investment efficiency, which indicates how efficiently the vendor can enhance the reliability of their software through investments in bug-fixing, testing, and other reliability-enhancing measures. Social welfare, calculated as the sum of software end users' surplus, the third-party vendor's payoff, and the platform's payoff, is not always enhanced through BBPs and depends on several factors including the vendor's reliability investment efficiency.

Another theme is the impact of bug bounty programs on software vendors, their motivations, and the vulnerability disclosure process. Ahmed et al. synthesized the literature on vulnerability disclosure mechanisms, including non-market and market mechanisms. They suggested that market mechanisms such as BBPs give vendors more control over the vulnerability disclosure process (Ahmed, Deokar, and Lee 2021). Zhou & Hui found that BBPs are not universally beneficial for all firms. They are advantageous for firms with low in-house efficiency in identifying vulnerabilities or those facing a high proportion of cooperative hackers who can be incentivized at a reasonable cost to report bugs to the firm rather than maliciously exploit the bug (J. Zhou and Hui 2020). Relatedly, Zhou and Hui found that implementation of the Internet Bug Bounty program disincentivized in-house contributors to work on bug reporting and other tasks, leading to a decline in their contributions, possibly due to increased competition from crowd contributors and decreased opportunities for inter-task learning (J. Zhou and Hui 2022).

The literature also explores the effects of external factors like the COVID-19 pandemic on BBPs. Zrahia et al. studied the impact of the COVID-19 shock on the

supply and demand dynamics of the bug bounty platform Bugcrowd. They found that while the supply of ethical hackers increased considerably during the pandemic, the demand for their services did not rise proportionately. These changes led to an increase in duplicate valid submissions and a decrease in the expected reward for valid submissions (Zrahia et al. 2022).

While the popularity of BBPs continues to rise and the literature has studied several aspects of BBPs as summarized earlier, their impact on software security warrants further scrutiny. A critical question, unaddressed in the extant literature, is how BBPs influence vendors' motivations regarding secure software releases, i.e., whether vendors with a BBP will release software earlier than they would have without a BBP. Relatedly, by identifying and mitigating vulnerabilities post-release, BBPs may inadvertently incentivize vendors to launch buggier products earlier, assuming that residual bugs can be addressed later. This potential moral hazard, where BBPs enable vendors to rush releases under the assumption that ethical hackers affiliated with BBPs will later identify vulnerabilities, could paradoxically erode producer surplus. The downstream costs of mitigating bugs discovered post-release – including those exploited by malicious hackers – may outweigh any temporary revenue gains from premature product launches. In this paper, we analyze the impact of BBPs on software release timing and producer surplus. In addition, we also consider the impact of bounty amounts on hackers' incentives. Finally, taking the number of malicious hackers as given, we characterize the optimal number of ethical hackers that should be in a BBP for a particular software.

Table 1: Abbreviations, Subscripts, and Variables

Abbreviations

WHH, eWHH, neWHH BHH BBP	White hat hackers, expert WHH, non-expert WHH respectively Black hat hackers Bug bounty program
SV Concerct Variables	Software vulnerability
General Variables	Number of MATHI
n	Number of eWHH
m	Number of BHH Number of neWHH
l	
t Effort Variables	Release time chosen by software vendor
α_{is}^{i} , α_{ins}^{i}	Effort of i th eWHH to find severe or non-severe bugs respectively
α_{s}, α_{ns}	Effort of remaining $(n - 1)$ eWHH to find severe or non-severe bugs respectively
β _{ins}	Effort of i th neWHH to find non-severe bug
β_{ns}	Effort of the remaining $(n - 1)$ neWHH to find non-severe bugs
μ_{is}	Effort of ith BHH to find non-severe bug
μ _s	Effort of the remaining $(n - 1)$ BHH to find severe bugs
Probability or Likeliho	ood Variables
$K_{s}(t), K_{ns}(t)$	Likelihood of the presence of a residual severe or non-severe bug at release time t
\mathbb{P}^{s}_{ie}	Probability that an eWHH finds a severe bug first
\mathbb{P}_{ine}^{ns}	Probability that an neWHH finds a non-severe bug first
\mathbb{P}^{s}_{ib}	Probability that a BHH finds a severe bug first
Revenue, Costs, Profit,	and Payoff Variables
c _w , c _b	Severity adjusted effort cost multiplier for eWHH and BHH respectively
p _s , p _{ns}	Monetary reward offer from software vendor to WHH for finding severe or non-severe bug respectively
r _s	Reputational gain from finding a severe bug, leading to career advancement
W	Illicit gain for a BHH when finding a severe bug first
R _{ie} , R _{ine} , R _{ib}	Expected payoff functions for eWHH, neWHH, and BHH
TC _s , TC _{ns}	Cost to a software vendor is BHH is first to find a severe or non-severe bug respectively.
R(t)	Revenue of the software vendor if it releases software at time t
П, П _b , П _{nb}	Expected profit of a software vendor (general, with BBP, without BBP)

3 Model Development

To motivate our model of the strategic interaction between the software vendor and the hackers, we first describe the players, their strategic variables, and the game's overall structure. We have three types of players: software vendors, white hat hackers (WHHs), and black hat hackers (BHHs). Software vendors aim to release their software as early as possible while mitigating the risks of software vulnerabilities (SVs). BBPs are a vital part of risk mitigation as they allow software vendors to attract WHHs to find bugs in the vendor's post-release software and coordinate disclosure of discovered bugs. The primary risk to the software vendor arises from BHHs, malicious actors exploiting software vulnerabilities for criminal gains. Software vendors choose the optimal release time and the optimal monetary reward they are willing to offer to WHH for finding SVs. The hackers, WHHs and BHHs, choose the optimal effort they will invest in finding the SVs.⁶

We model the strategic interaction between the players as a two-stage game. In the first stage, the software vendor chooses the release time and the bounty amounts. In the second stage, WHHs and BHHs simultaneously choose how much effort they should exert to find SVs after observing the actions selected by the software vendor in the first stage. To obtain subgame perfect equilibrium, we solve the game by backward induction, first finding the equilibrium behavior of hackers in the second stage. The software vendor acts as a Stackelberg player by incorporating the

⁶ The total effort invested for WHHs is finding and reporting the SVs, whereas for BHHs it is finding and exploiting the SVs.

anticipated behavior of the hackers in making his choice in the first stage of the game.

3.1 Severity of Software Vulnerability

An essential consideration for SVs is their severity. Severity has been defined as "the highest failure impact that the defect could (or did) cause, as determined by (from the perspective of) the organization responsible for software engineering" (IEEE Computer Society 2010). Thus, severity plays a significant role in the management of discovered vulnerabilities. Munaiah and Meneely state that vulnerability severity metrics serve three key purposes: (a) inform users about the potential impact of a vulnerability, (b) help vendors in triaging the resolution of a vulnerability, (c) facilitate analysts in their analyses of vulnerabilities (Munaiah and Meneely 2016). Although many software vulnerability metrics exist, one of the most widely used metrics is the Common Vulnerability Scoring System (CVSS), developed and maintained by FIRST (Forum of Incident Response and Security Teams). CVSS provides numerical scores (0.0—10.0), which can be mapped to qualitative ratings of increasing severity as none, low, medium, high, and critical (FIRST 2023). The final two categories, high and especially *critical*, naturally garner heightened attention from all stakeholders, including users, vendors, cybersecurity entities, ethical hackers, malicious hackers, and others. For our analysis, we classify software vulnerabilities as "severe" (mapping to high and critical in the CVSS qualitative ratings) and "non-severe."

As discussed earlier, WHHs find and disclose SVs, which leads to vendors fixing the vulnerability through a patch. While talent plays a role in becoming an effective WHH, the skills and capabilities developed through study and experience are often equally, if not more, important. We distinguish the expertise of WHHs by classifying them into two types: expert white hat hackers (eWHH) and non-expert white hat hackers (neWHH). Black hat hackers (BHH) or malicious hackers work outside legal and ethical bounds to find and exploit software vulnerabilities for illicit gains. For example, a BHHs may exploit a vulnerability to illegally access and encrypt a vendor's data and demand a ransom to make the data accessible again. While BHHs may also exist at various skill levels, we are primarily concerned with technically sophisticated BHHs, who have the talent, skills, and capabilities to cause damage without getting caught by law enforcement agencies. In summary, we classify hackers into one of three types, eWHH, neWHH, and BHH. Further, we assume that *n* eWHH, l neWHH, and m BHHs are simultaneously trying to find SVs (<u>Table 1</u>) summarizes the abbreviations and variables we use in our discussion).

3.2 Strategic Interaction Between BHHs and WHHs

We first analyze the strategic interaction between BHHs and WHHs, which constitutes the second stage of our 2-stage game structure.

As mentioned earlier, hackers' strategic choice is their level of effort to find SVs. Hackers incur a cost for the efforts. We consider the cost function of an individual hacker, *i* to be a quadratic function of effort level. Let $\alpha_{is} \in (0, 1)$, $\beta_{is} \in (0, 1)$, $\mu_{is} \in (0, 1)$ be the effort of the *i*-th eWHH, neWHH, and BHH, respectively, to find severe SVs (and α_{ins} , β_{ins} , μ_{ins} for non-severe SVs). Since eWHHs spend effort on both severe and non severe EVs, their cost is the sum of the costs for the effort spent on both severe and non-severe SVs:

$$F_e = \frac{c_w \alpha_{is}^2}{2} + \frac{\alpha_{ins}^2}{2} + \alpha_{is} \alpha_{ins} \text{ where } c_w > 1$$

The severity-adjusted effort cost multiplier for eWHH, denoted as " c_w " amplifies the effort cost for detecting severe software vulnerabilities, reflecting the heightened complexity and resource investment required in these cases. The interaction term, $\alpha_{is} \times \alpha_{ins}$, reflects the additional cost of working concurrently on both severe and non-severe SVs. If an eWHH invests more effort in severe SVs, investing time in non-severe SVs becomes more expensive, and vice versa.

For simplicity, we assume that due to the lack of expertise of the neWHH, the cost incurred to allocate effort to severe bugs is infinitely expensive for them. Hence, neWHHs do not allocate any effort to severe bugs, i.e., $\beta_{is} = 0$. The cost for the neWHHs is:

$$F_{ne} = \frac{\beta_{ins}^2}{2}.$$

Finally, BHHs do not allocate any effort to find non-severe SVs, i.e., $\mu_{ins} = 0$ because of very small or no illicit gains from non-severe SVs. The costs for BHHs is thus:

$$F_b = \frac{c_b \mu_{is}^2}{2}.$$

The severity-adjusted effort cost multiplier for BHH, $c_b > 1$, amplifies the effort cost for detecting severe software vulnerabilities.

WHHs' payoffs have two primary components: First, they may receive a monetary reward (bounty) for reporting a previously unknown SV. Second, they may accrue gains in reputation that may lead to career advancement for reporting a previously unknown SV. For eWHHs' payoffs, we assume that reporting severe SVs leads to both monetary and reputational gains, but reporting non-severe SVs leads only to monetary gains without any reputational gains.⁷ Furthermore, the monetary gains for non-severe SVs are smaller than those for severe SVs. Thus, if an eWHH is the first to report a severe SV, the eWHH receives a payoff $r_s + p_s$, where r_s is the reputational gain that leads to career advancement, and $p_{\rm g}$ is the bounty paid by the BBP. On the other hand, if an eWHH is the first to report a non-severe SV, he receives a payoff p_{ns} but does not accrue any gains in reputation. For neWHHs, as the effort to find severe bugs, $\beta_{is} = 0$, their payoff accrues only from rewards for non-severe bugs. Thus, when a neWHH reports a previously unknown non-severe SV, he receives the payoff p_{nc} .

⁷ Our reputation measure focuses on the technical prowess of the hackers, which is associated with their ability to discover complex and impactful bugs. BPPs may have a more nuanced definition of reputation. For instance, a prominent BPP platform, HackerOne, describes reputation as "your reputation measures how likely your finding is to be immediately relevant and actionable. Reputation is points gained or lost based on report validity. It's weighted based on the size of the bounty and the criticality of the reported vulnerability. Reputation is based exclusively on your track record as a hacker" (HackerOne 2023). Despite the emphasis on report validity in HackerOne's reputation definition, both definitions are broadly similar.

BHHs' payoffs accrue from exploiting an SV for illicit gains such as ransom from affected parties in a ransomware attack. These illicit gains are only likely to accrue from severe bugs. Thus, we assume BHHs do not allocate any effort to find non-severe SVs and further assume that their payoff "*W*" accrues from exploiting severe SVs. For example, BHHs may gain *W* by selling sensitive data such as credit card numbers to criminals or charging ransom from a victim to regain access to their data.

3.2.1 Success Probabilities under Type-Symmetric Equilibrium

Although we focused on the cost and payoff functions of an individual hacker of each type (eWHH, neWHH, BHH) in the previous subsection, software products are simultaneously evaluated by multiple hackers of each type to discover SVs. As assumed earlier, there are n eWHHs and l neWHHs in the BBP trying to find SVs. Concomitantly, there are m BHHs trying to find SVs.

For a hacker of any type, we define "success" as being the first to discover the SV. From the point of view of individual WHH, it only matters if an individual WHH is the first to discover the SV, as the "bounty" is only paid to the first reporter of the SV. For simplicity, we also assume that BHHs benefit if they are the first to discover the SV. Although it is plausible that multiple BHHs can accrue illicit gains from the same SV, the alternative is arguably more plausible. For instance, after one BHH exploits an SV for gains, the software vendor may learn about and fix the SV so it cannot be exploited again.

To make the subsequent analysis tractable, we focus on the derivation of a type-symmetric equilibrium in which hackers of the same type (eWHH, neWHH, BHH) use the same strategy, i.e., hackers of the same type choose the same level of effort for SVs. Under this assumption, the success probability changes based on the effort levels of hackers. Consider a focal eWHH "*i*" who invests effort α_{is} in finding severe SVs. The average effort of the other (n - 1) eWHHs and *m* BHHs who devote effort to finding severe SVs is then given by $\frac{(n-1)\alpha_s + m\mu_s}{n+m-1}$, where α_s and μ_s are the effort levels selected by the two types at the type-symmetric equilibrium. If all n eWHHs and *m* BHHs use the same level of effort for severe SVs, the base probability of a given hacker to discover a bug first is inversely related to the number of hackers, (n + m) searching for bugs. To keep the subsequent analysis tractable, we assume that this base probability is $\frac{1}{n+m}$.⁸ However, a focal hacker can increase the odds of being the first to find a severe bug, above this base probability, by allocating more effort than others for the same type of bug. If the focal eWHH *i*, increases the effort then the increased probability of finding the bug is the difference between the effort, $\alpha_{i,i}$, allocated by focal eWHH *i*, and the average effort allocated by all other hackers

searching for severe bugs, $\frac{(n-1)\alpha_s + m\mu_s}{n+m-1}$. Thus, the probability for eWHH *i* to find a severe bug first is:

$$\mathbb{P}_{ie}^{s} = \frac{1}{n+m} + \frac{1}{n+m} \left(\alpha_{is} - \frac{(n-1)\alpha_{s} + m\mu_{s}}{n+m-1} \right)$$
(1)

⁸ This probability is analogous to the probability that a bidder will win a second price auction (1/N) when all "N" bidders have uniformly and identically distributed valuations.

Similarly, the success probabilities for eWHH *i* finding non-severe SV (\mathbb{P}_{ie}^{ns}), neWHH finding non-severe SV (\mathbb{P}_{ine}^{ns}), and BHH finding a severe SV (\mathbb{P}_{ib}^{s}) are given by:

$$\mathbb{P}_{ie}^{ns} = \frac{1}{n+l} + \frac{1}{n+l} \left(\alpha_{ins} - \frac{(n-1)\alpha_{ns} + l\beta_{ns}}{n+l-1} \right)$$
(2)

$$\mathbb{P}_{ine}^{ns} = \frac{1}{n+l} + \frac{1}{n+l} \left(\beta_{ins} - \frac{n\alpha_{ns} + (l-1)\beta_{ns}}{n+l-1} \right)$$
(3)

$$\mathbb{P}_{ib}^{s} = \frac{1}{n+m} + \frac{1}{n+m} \left(\mu_{is} - \frac{n\alpha_{s} + (m-1)\mu_{s}}{n+m-1} \right)$$
(4)

In our primary analyses, we use this additive formulation for success probabilities. We also explored a multiplicative formulation in which the base probability is scaled up or down by the ratio of the effort between the focal hacker and the average effort of the rest of the hackers. These analyses are included in <u>Appendix A</u> and strengthen the results we report in this paper.

3.2.2 Optimal Effort of the Hackers

Hackers' expected payoff depends on their success probabilities, their payoffs upon success,⁹ the cost of their efforts, and the likelihood of residual severe and non-severe SVs existing at the time of software release. The vendor makes efforts to release products with fewer vulnerabilities. However, it is well-established that

⁹ Bounties in the case of WHH and illicit gains in the case of BHH.

reducing the number of bugs takes time (Arora, Caulkins, and Telang 2006). The longer a vendor takes to release a product because of additional testing, the fewer vulnerabilities it is likely to have. We designate by $K_{s}(t)$ and $K_{ns}(t)$ the likelihood of residual severe and non-severe SVs existing at release (i.e., SVs that have not been detected and fixed in pre-release software testing.) If software release is delayed to a later а longer testing period reduces residual SVs. time, Hence, $K_{s}'(t) < 0$ and $K_{ns}'(t) < 0.^{10}$ We assume that the rate of decrease slows down with time, i.e., $K_{s}''(t) > 0$. In the second stage of the game, the hackers take the software vendor's choice of the bounty rewards $(p_s \text{ and } p_{ns})$ and the release time, which affects $K_{s}(t)$ and $K_{ns}(t)$, as given, and choose the optimal effort in response.

We can formulate the total expected payoff of the focal eWHH *i*, R_{ie} as:

$$R_{ie} = K_s(t)\mathbb{P}_{ie}^s\left(r_s + p_s\right) + K_{ns}(t)\mathbb{P}_{ie}^{ns}\left(p_{ns}\right) - F_{ie}$$

The first and second terms on the RHS are the expected payoffs from finding severe and non-severe SVs respectively. The last term is the effort cost of finding the SVs. Substituting the expressions for success probabilities and the effort costs, we obtain:

$$R_{ie} = K_{s}(t) \frac{1}{n+m} \left[1 + \left(\alpha_{is} - \frac{(n-1)\alpha_{s} + m\mu_{s}}{n+m-1} \right) \right] \left(r_{s} + p_{s} \right) +$$
(5)

¹⁰ We claim that strict inequality is a realistic assumption as even with large investments in software testing and a prolonged testing period, the vendor cannot guarantee that no SV remains that could potentially be discovered with more testing.

$$K_{ns}(t) \frac{1}{n+l} \left[1 + \left(\alpha_{ins} - \frac{(n-1)\alpha_{ns} + l\beta_{ns}}{n+l-1} \right) \right] (p_{ns}) - \left[\frac{c_w \alpha_{is}^2}{2} + \frac{\alpha_{ins}^2}{2} + \alpha_{is} \alpha_{ins} \right]$$

Similarly, the objective functions for neWHH and BHH are given by:

$$R_{ine} = K_{ns}(t) \frac{1}{n+l} \left[1 + \left(\beta_{ins} - \frac{n\alpha_{ns} + (l-1)\beta_{ns}}{n+l-1} \right) \right] (p_{ns}) - \frac{\beta_{ins}^{2}}{2}$$
(6)

$$R_{ib} = \frac{K_s(t)W}{n+m} \left[1 + \left(\mu_{is} - \frac{n\alpha_s + (m-1)\mu_s}{n+m-1} \right) \right] - \frac{c_b \mu_{is}^2}{2}$$
(7)

Note that the hackers' objective function is a concave function of their decision rules $(\alpha_{is'} \alpha_{ins}, \beta_{ins'} \mu_{is})$, so first-order conditions are sufficient for their maximizations. In a type-symmetric equilibrium, the eWHH may allocate effort to reporting severe and non-severe SVs (i.e., $\alpha_{is} > 0$ and $\alpha_{ins} > 0$) or to reporting only severe bugs (i.e., $\alpha_{is} > 0$ and $\alpha_{ins} = 0$). While eWHHs who work on both severe and non-severe vulnerabilities exist, anecdotally, the second case seems more prevalent among expert bounty hunters, circa 2023. Expert bounty hunters are not only attracted by the higher rewards for severe SVs but also by the challenge of finding technically complex hacking attacks that unearth severe bugs. Additionally, severe SVs are more likely to get adjudicated swiftly by BBP for bounty rewards. Thus, we will relegate the analysis of the first case (i.e., $\alpha_{is} > 0$ and $\alpha_{ins} > 0$) to Appendix B and focus our discussion in the paper on the more important, second case (i.e.,

 $\alpha_{is} > 0$ and $\alpha_{ins} = 0$). This case arises in the equilibrium if $\frac{K_s(t)(r_s + p_s)}{(n+m)c_w} > \frac{K_{ns}(t)p_{ns}}{n+l}$, namely if the expected payoff of eWHH from severe bugs (normalized by the effort

cost multiplier) is greater than the expected payoff from non-severe bugs.

Optimizing the hackers' objectives yields the following effort levels chosen at the symmetric equilibrium.

$$\alpha_{is} = \alpha_s = \frac{1}{(n+m)c_w} K_s (t) (r_s + p_s), \ \alpha_{ins} = 0$$
(8)

$$\beta_{ins} = \beta_{ns} = \frac{1}{l} K_{ns} \ (t)(p_{ns}) \tag{9}$$

$$\mu_{is} = \mu_s = \frac{1}{c_b(n+m)} K_s(t)(W)$$
(10)

Hence, a given hacker exerts more effort if the likelihood of residual SVs in the software on which she is working is higher, the reward she expects is higher, the number of competing hackers is smaller, and the cost of effort is lower (the severity-adjusted effort cost multipliers, c_w and c_b , are smaller for eWHH and BHH, respectively).

Substituting the symmetric effort level chosen by each type (α_s , β_{ns} , and μ_s) into the expressions of the probabilities of success derived earlier, we obtain the following probabilities of success by an individual hacker at the symmetric equilibrium:

$$\mathbb{P}_{ie}^{s} = max(0, \frac{1}{n+m} \left[1 + \frac{mK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{(r_{s}+p_{s})}{c_{w}} - \frac{W}{c_{b}} \right\} \right]$$
(11)

$$\mathbb{P}_{ine}^{ns} = \frac{K_{ns}(t)p_{ns}}{l} \tag{12}$$

$$\mathbb{P}_{ib}^{s} = max(0, \frac{1}{n+m} \left[1 + \frac{nK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{W}{c_{b}} - \frac{(r_{s}+p_{s})}{c_{w}} \right\} \right]$$
(13)

Hence, a WHH (BHH) is more (less) likely to succeed if the reward the WHH expects discounted by his cost of effort $\frac{(r_s+p_s)}{c_w}$ is higher and if the gain derived by a BHH from exploiting vulnerabilities discounted by his cost $\frac{W}{c_b}$ is smaller, respectively. Success is also more likely if the likelihood of residual vulnerabilities is higher, as reflected by bigger values of $K_s(t)$ and $K_{ns}(t)$. As the number of competing hackers searching for a given type of bug increases, the success probability of one given hacker decreases.

Figure 1 illustrates that the success probability for a WHH increases and that of a BHH decreases when the bounty offered to WHH is bigger.



Figure 1: Success Probability for WHH & BHH vs. Bounty for SV Offered to WHH

A more generous bug bounty incentivizes WHHs to exert more effort, thus raising the WHHs' success probability and reducing BHHs' success probability.¹¹ We state this observation in the following proposition:

Proposition 1: In the context of a bug bounty program (BBP), as the monetary reward (bounty) offered to white-hat hackers (WHH) for discovering severe bugs increases, (a) the effort invested by WHH in searching for bugs increases, and (b) the probability that a WHH will find a severe bug first increases, while the probability that a black-hat hacker (BHH) will find a severe bug first decreases..

3.3 Strategic Behavior of the Software Vendor

We now consider the strategic behavior of the software vendor in the first stage of our 2-stage game. The software vendor chooses the release time and the bounty award to optimize his payoff, considering the likely response of the hackers in the second stage of the game.

When choosing the time of software release, vendors consider the tradeoff between releasing software early to gain revenue R(t) and delaying the release to allow for more testing to reduce the risk of post-release discovery of vulnerabilities. The likelihood of residual vulnerabilities depends on the amount of testing conducted before release. More testing leads to higher quality software with fewer latent bugs but delays time-to-market. Thus, release time t is an important strategic variable for

¹¹ In Appendix A, we reformulated the success probability as discussed earlier. In this new formulation, increases in bounty introduces a dual effect of concurrently incentivizing the WHHs to increase efforts and the BHHs to reduce efforts. This dual effect on effort is stronger than the single effect we obtain here.

any software vendor. Delaying the software's release reduces its revenues but also reduces the number of severe and nonsevere bugs, $K_s(t)$ and $K_{ns}(t)$, that can be discovered and potentially exploited. We designate by TC_s the vendor's cost if a severe SV occurs and a BHH succeeds in finding it first. We further designate by TC_{ns} the vendor's cost if a non-severe SV occurs and no WHH finds it, but rather a user finds it, where $TC_s \gg TC_{ns}$.

When a software vendor participates in a BBP, it chooses the bounty amounts p_s and p_{ns} , and the time of release *t*, to maximize its profit:

$$\Pi = R(t) - K_{s}(t) \left(m \mathbb{P}_{b}^{s} \right) \left(TC_{s} \right) - K_{s}(t) \left(n \mathbb{P}_{e}^{s} \right) \left(p_{s} \right) - K_{ns}(t) \left(l \mathbb{P}_{ne}^{ns} \right) \left(p_{ns} \right) - K_{ns}(t) \left(TC_{ns} \right) \left[1 - l \mathbb{P}_{ne}^{ns} \right]$$

$$(14)$$

Note that in the cost terms (the 2nd, 3rd, 4^{th,} and 5th RHS terms), we scale the individual success probabilities of hackers of a particular type by the number of hackers of the corresponding type, as the vendor will incur a cost if any one of these hackers finds the SVs. Term 2 is the cost incurred by the vendor when one of the BHHs finds the vulnerability first, and term 3 is the cost incurred in paying bounties for severe bugs. Cost term 4 is the cost incurred for paying bounties for non-severe bugs, and cost term 5 relates to non-severe SVs discovered by software users, requiring the vendor to fix the SVs. Plugging in the expressions for success probabilities into the vendor's objective function, we obtain:

$$\Pi = R(t) - K_{s}(t) \frac{m}{n+m} \left[1 + \frac{nK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{W}{c_{b}} - \frac{(r_{s}+p_{s})}{c_{w}} \right\} \right] (TC_{s}) - K_{s}(t) \frac{n}{n+m} \left[1 + \frac{mK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{(r_{s}+p_{s})}{c_{w}} - \frac{W}{c_{b}} \right\} \right] (p_{s}) - (15)$$
$$(K_{ns}(t)p_{ns})^{2} - K_{ns}(t)TC_{ns} \left[1 - K_{ns}(t)p_{ns} \right]$$

The total realized revenue from the software declines as the release is delayed, i.e., R'(t) < 0. Moreover, we assume that the revenue fall is sharper when products are delayed further, namely R''(t) < 0.

3.3.1 Optimal Bounty Set by the Software Vendor

We start by optimizing the vendor's objective with respect to the monetary amounts awarded to white hats. It is easy to show that the objective is a concave function of p_s and p_{ns} , implying that first-order conditions are also sufficient. Optimizing with respect to p_s and p_{ns} , we obtain:

$$p_{s} = \frac{1}{2} \left[TC_{s} + \frac{c_{w}}{c_{b}} W - r_{s} \right] - \frac{1}{2} \frac{(m+n)(n+m-1)}{m} \frac{1}{K_{s}(t)} c_{w}$$

$$p_{ns} = \frac{TC_{ns}}{2}$$
(16)
(17)

The software vendor sets a larger monetary reward (
$$p_s$$
) for severe SVs if (i) the vendor's costs (TC_s) are high when a BHH succeeds in discovering severe SV, (ii) the BHH's illicit gains W are high when the BHH succeeds in discovering severe SV, or when the cost of effort c_b incurred by the BHH to find severe bugs is low, (iii) the likelihood that a severe SV exists at release time $K_s(t)$ is high. On the other hand, the

software vendor sets a smaller monetary reward if the reputational gains of WHH, r_s are high.

When facing high costs from BHH exploitation or a high likelihood of SV presence at release, the vendor offers higher monetary rewards to entice top eWHHs to identify SVs in their software before BHHs can. Similarly, if the illicit gains to the BHH are high or when their cost of effort is low, they are highly motivated in their attack, so the vendor has to counter by raising the monetary reward to attract eWHHs. In contrast, if the reputational gains for eWHHs are high when discovering severe bugs, the vendor does not have to set large monetary rewards to attract eWHHs. The cost of effort of eWHHs has an ambiguous effect on the bug bounty. On one hand, high effort cost implies that the bounty should be higher to allow eWHHs to cover their higher cost. On the other hand, high effort cost implies that eWHHs are less likely to find severe SVs, and therefore, there is less reason to attract them to participate in the BBP.

For a bounty program to exist it should be that the vendor is willing to pay positive bounties to WHHs. In addition, it should also be that WHHs and BHHs have positive probabilities of finding SVs first. Using the expressions we derive for the optimal bounties in (16) and (17) and the probabilities in (11)-(13), we can obtain conditions that the parameters of the model satisfy to support the existence of a bounty program. Condition 1 reports the requirements that the parameters should satisfy.

Condition 1:
$$LB < \left[\frac{W}{c_b} - \frac{r_s}{c_w}\right] < UB$$
, where
 $LB \equiv max \left\{ \frac{(m+n)(n+m-1)}{mK_s(t)} - \frac{TC_s}{c_w} \right\}, \frac{TC_s}{c_w} - \frac{(2m+n)(m+n)(n+m-1)}{mnK_s(t)} \right\}$ and
 $UB \equiv \frac{(m+n)(n+m-1)}{mK_s(t)} + \frac{TC_s}{c_w}.$

The upper bound is always bigger than the lower bound implying that there is a nonempty region for the parameters (i.e., the difference $\left[\frac{W}{c_b} - \frac{r_s}{c_w}\right]$) that supports the existence of the bounty program ($p_s > 0$) and positive success probabilities for hackers.

3.3.2 Optimal Release Time without BBP

To determine the effect of establishing a bounty program on the release time of the software, we start by considering the optimal release time of the software for a vendor that does not establish such a program. First, note that in the absence of a bounty program, $p_s = 0$, and the success probabilities of eWHH and BHH change accordingly:

$$\mathbb{P}_{ie}^{s} = max(0, \frac{1}{n+m} \left[1 + \frac{mK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{r_{s}}{c_{w}} - \frac{W}{c_{b}} \right\} \right]$$
(18)

$$\mathbb{P}_{ib}^{s} = max(0, \frac{1}{n+m} \left[1 + \frac{nK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{W}{c_{b}} - \frac{r_{s}}{c_{w}} \right\} \right]$$
(19)

There are potentially three types of costs when SVs are discovered. The vendor incurs the cost TC_s if a severe SV is discovered and exploited by a BHH. It may incur

the cost xTC_s with $x \in (0, 1)$ when an eWHH discovers a severe SV and potentially discloses it publicly without coordination with the vendor. This uncoordinated disclosure imposes an externality on the software vendor and potentially on the users of its software. We assume that this cost, xTC_s , is not as high as the damage inflicted by a BHH who finds the severe SV first (i.e., $x \in (0, 1)$). Finally, the vendor may also incur the cost TC_{ns} if a nonsevere SV is discovered by users,¹² which may hurt the vendor's reputation.

The objective function of the vendor without a bounty program becomes:

$$\Pi_{nb} = R(t) - K_s(t)m\mathbb{P}_{ib}^s TC_s - K_s(t)n\mathbb{P}_{ie}^s(xTC_s) - K_{ns}(t)TC_{ns}$$
(20)

Since both \mathbb{P}_{ib}^s and \mathbb{P}_{ie}^s are strictly positive, the first-order condition to determine the optimal release time is:

$$\frac{\partial \Pi_{nb}}{\partial t} = R'(t) - K'_{s}(t) \frac{m}{n+m} \left[1 + \frac{2nK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{W}{c_{b}} - \frac{r_{s}}{c_{w}} \right\} \right] TC_{s} - K_{s}'(t) \frac{n}{n+m} \left[1 + \frac{2mK_{s}(t)}{(n+m-1)(n+m)} \left\{ \frac{r_{s}}{c_{w}} - \frac{W}{c_{b}} \right\} \right] (xTC_{s}) - K_{ns}'(t) TC_{ns} = 0$$
(21)

The assumptions that R''(t) < 0 and K''(t) > 0 ensure that this first-order condition is also sufficient, given that the objective is a concave function of t in this

¹² We assume that novice users do not have the expertise and incentive to exert the effort necessary to find severe bugs. Nonsevere bugs are much easier to find, even by novice users.

case.¹³ From the first order condition, the optimal release time *t* increases as TC_{s} , TC_{ns} , *x*, and $K_{s}(t)$ increase. The vendor will delay release if the cost of SV exploitation by BHH is high, if the probability of BHH exploitation due to uncoordinated disclosure by WHH is high, or if the likelihood of residual SV at release time is high. In these situations, the software vendor will delay the release to test the software longer.

3.3.3 BBPs Guarantee Higher Profits for Software Vendors

For a bug bounty program to exist, it must be that the monetary rewards to the white hats are positive (i.e., $p_s > 0$ and $p_{ns} > 0$). Assuming that Condition 1 is satisfied, we can substitute the expressions obtained for the optimal monetary awards in the objective function of the vendor with BBP. This objective can then be expressed in terms of the objective of the vendor without BBP, as follows:

$$\Pi_{b} = \Pi_{nb} + \frac{mn(K_{s}(t))^{2}p_{s}^{2}}{(n+m-1)(n+m)^{2}c_{w}} + (K_{s}(t))^{2}p_{ns}^{2} + NK_{s}(t)xTC_{s}max(0, \frac{1}{n+m}\left[1 + \frac{mK_{s}(t)}{(n+m-1)(n+m)}\left\{\frac{r_{s}}{c_{w}} - \frac{W}{c_{b}}\right\}\right])$$
(22)

Proposition 2 follows directly from the above expression.

Proposition 2: Let Π_{b} , Π_{nb} denote the expected profit function of a software vendor with and without a bug bounty program, respectively. If the vendor finds it optimal to award positive monetary awards to white hat hackers, then $\Pi_{b} > \Pi_{nb}$.

¹³ These conditions are stronger than necessary. In our analysis, we will assume that concavity holds, so that first order conditions are sufficient.

The Proposition's finding explains the large and growing number of bug bounty programs. The overall cost of bugs first found by BHH (TC_s) is extremely high. This cost includes not only the monetary damage inflicted by hackers but also the regulatory and reputational harm incurred by the vendor. Software vendors would thus opt into BBPs to reduce such costs.

3.3.4 Release Time with BBP

To obtain the optimal release time for a vendor with BBP, we maximize the objective function of the vendor with a bounty program (Equation 22) with respect to the release time, *t*:

$$\frac{\partial \pi_{b}}{\partial t} = \frac{\partial \pi_{nb}}{\partial t} + \frac{\partial \left[\frac{mn(K_{s}(t))^{2}P_{s}^{2}}{(n+m-1)(n+m)^{2}c_{w}} + \left(K_{s}(t)\right)^{2}P_{ns}^{2} + \frac{nK_{s}(t)xTC_{s}}{n+m}\left[1 + \frac{mK_{s}(t)}{(n+m-1)(n+m)}\left\{\frac{R_{s}}{c_{w}} - \frac{W}{c_{b}}\right\}\right]\right]$$
(23)

Denote $D_{(t,\Pi)} = \frac{\partial \Pi_b}{\partial t} - \frac{\partial \Pi_{nb}}{\partial t}$

$$\Rightarrow D_{(t,\Pi)} = \frac{\partial}{\partial t} \left[\frac{mn(K_s(t))^2 P_s^2}{(n+m-1)(n+m)^2 c_w} + (K_s(t))^2 P_{ns}^2 + \frac{nK_s(t)xTC_s}{n+m} \left[1 + \frac{mK_s(t)}{(n+m-1)(n+m)} \left\{ \frac{R_s}{c_w} - \frac{W}{c_b} \right\} \right] \right]$$
(24)

Evaluating $\frac{\partial \pi_b}{\partial t}$ at the optimal time chosen by a software vendor without BBP, t_{nb}^* , yields that the first term $\frac{\partial \pi_{nb}}{\partial t} = 0$ and the sign of $D_{(t,\Pi)}$ determines whether the

release time of the vendor with BBP is earlier or later than that of the vendor without BBP. That is, given the concavity of the objective function $\pi_{b'}$ if $D_{(t,\Pi)}$ is negative, a software vendor with BBP releases its software earlier than a vendor without BBP. Given the expressions derived for, P_s and $P_{ns'}$, it is easy to show that this derivative is indeed negative because $\frac{\partial P_s}{\partial t} < 0$, $K_{s'}(t) < 0$, and $K_{ns'}(t) < 0$.¹⁴ Hence, the software vendor with BBP will definitely release the software earlier. We state this result in Proposition 3.

Proposition 3: The release time of software of a vendor with a bug bounty program is earlier than that of a vendor without such a program.

3.3.5 Optimal Number of WHH in BPP

We now consider "*n*," the number of eWHHs in a program, to be a vendor's choice variable and characterize the optimal number of eWHHs that should participate in a BBP. Such a choice applies primarily to private BBPs. A private BBP allows a vendor to choose the eWHHs that the vendor will allow to participate in its BPP. Thus *private* BPPs not only allow the vendor to choose the quality of eWHH participants but also the number of such participants. Under the assumption that the objective function is concave in "*n*," the solution to the first order condition will yield the optimal value for "*n*." Differentiating the objective function by "*n*" yields the expression¹⁵:

¹⁴ The inequality indicating that the optimal bounty offered for SV declines as software is released later follows from the derivative of the optimal bounty expression.

¹⁵ Note that the second derivative of the objective with respect to *n* is negative, guaranteeing that assumed concavity of the objective with respect to *n*. Hence, the first order condition is also sufficient.

$$\frac{\partial \pi_{b}}{\partial n} = \frac{mK_{s}^{2}(t)(C_{s}-p_{s})^{2}}{c_{w}(n+m)^{2}} \left[-2n^{2} - n(m-1) + m(m+1) \right]$$
(25)

Solving the first order condition, $\frac{\partial \pi_b}{\partial n} = 0$, for "*n*" yields the following solution:

$$n = \frac{\sqrt{9m^2 - 10m + 1}}{4} - \frac{(m - 1)}{4} \tag{26}$$

The solution has three implications: (i) the optimal choice of "*n*" depends only on the number of BHHs that are likely to attack the software, (ii) the optimal number of eWHHs, "*n*," is smaller than the number of "*m*," (ii) $\frac{\partial n}{\partial m} > 0$, i.e., the optimal number of eWHHs that the vendor allows into the private program increases as the expected number of "*m*" increases. Thus, a software vendor that releases very prominent software (e.g., widely used web browser) that will likely attract a large number of BHHs should optimally have a larger number of eWHHs in its private BPP than a vendor with less prominent software.

4 Conclusion

Our paper highlights three key findings that shed light on the growing adoption of bug bounty programs (BBPs) by software vendors. First and foremost, our results demonstrate that under reasonable assumptions, a software vendor can increase its expected profit by participating in a BBP. This profitability advantage explains the rising popularity of BBPs among software vendors and the success of BBP platforms like HackerOne. Second, our model suggests that software vendors with BBPs will likely release their products earlier than vendors without such programs. As testing for vulnerabilities and fixing them is a time-consuming process, an earlier release implies that vendors with BBPs may release software with a relatively higher number of bugs. While this could potentially increase the risk for software users, the vendor can mitigate this risk by leveraging the BBP, where ethical hackers are contractually obligated to coordinate the disclosure of vulnerabilities they find, allowing the vendor to address them promptly if it chooses. The software vendor certainly mitigates the risks to its market reputation by controlling the disclosure of vulnerabilities through BBPs.

Third, if a vendor has the ability to choose the number of ethical hackers to invite into its BBP, the optimal number depends solely on the expected number of malicious hackers likely to seek vulnerabilities for exploitation. Notably, the optimal number of ethical hackers is lower than the expected number of malicious hackers but increases as the expected number of malicious hackers increases.

Finally, higher bounties incentivize ethical hackers to exert more effort, thereby increasing the probability that they will discover severe vulnerabilities first while reducing the success probability of malicious hackers.

While our analysis clearly shows the profitability benefit of BBPs to software vendors, the overall social welfare impacts of BBPs require further analysis. Qualitatively, consumers gain earlier access to new software features through accelerated release cycles enabled by BBPs. However, this comes at the potential cost of receiving buggier, less secure software in the short term before vulnerabilities identified through bounties can be patched. Lack of transparency, as vendors control vulnerability disclosures, creates information asymmetries and further obfuscates security risks to consumers. Conversely, BBPs foster positive externalities by incentivizing ethical WHHs to contribute to improving software security, and mitigating cybercrime's societal burden over time while providing legal income opportunities in this field. BBPs also improve economic efficiency as skilled WHHs are not bound to work exclusively for a single software vendor but can freely use their skills across various software vendors. Nevertheless, unpatched vulnerabilities could briefly enable malicious hacking, generating negative externalities.

Overall, our findings collectively underscore the potential benefits of bug bounty programs for software vendors, not only in terms of improved profitability but also in enabling earlier product releases while managing the associated risks to their reputation through coordinated vulnerability disclosure. As the threat landscape evolves, the adoption of BBPs is likely to continue gaining momentum, providing software vendors with a valuable tool to enhance their security posture and stakeholder trust.

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Appendix A: Modified specification of probabilities of finding bug first

In this section, we modify the probability of an individual hacker to find a bug first, focusing on the case that expert white hats allocate effort only to finding severe bugs. We assume that the added probability of being first above the basic probability $(\frac{1}{m+n})$ for severe, and $\frac{1}{l}$ for non-severe) is formulated as a ratio of the individual hacker's effort and the average effort exerted by other hackers who allocate efforts to find the same type of bugs. This is in contrast to the assumption in the main text, where this added probability was formulated as a difference between the two. Whereas in the main text the optimal effort level selected by an individual hacker was independent of the choice of other hackers, in the new formulation this choice does depend on how much effort is exerted by others. We demonstrate, however, that our main results do not change significantly. Establishing a bounty program leads to reduced probability of black hats finding severe bugs.

The following are the payoff functions of the hackers with the new formulation.

Payoff of expert white hat:

$$R_{we} = \frac{K_{s}(t)(r_{s} + p_{s})}{n + m} \left[\frac{\alpha_{is}}{\left[(n-1)\alpha_{s} + m\mu_{s} \right] / (n + m - 1)} \right] - \frac{c_{w} \alpha_{is}^{2}}{2}, \text{ where } c_{w} > 1.$$

Payoff black hat:

$$R_{b} = \frac{K_{s}(t)W}{n+m} \left[\frac{\mu_{is}}{\frac{(n\alpha_{s}+(m-1)\mu_{s})}{(n+m-1)}} \right] - \frac{c_{b}\mu_{is}^{2}}{2}, \text{ where } c_{b} > 1.$$

Payoff of non-expert white hat:

$$R_{wne} = \frac{K_{ns}(t)p_{ns}}{l} \left[\left(\beta_{ins} / \beta_{ns} \right) \right] - \frac{\beta_{ins}^{2}}{2}$$

To illustrate, when the effort α_{is} of an individual expert white hat is bigger than the average effort $\frac{(n-1)\alpha_s + m\mu_s}{(n+m-1)}$ of other hackers working on finding severe bugs, his basic probability is bigger than $\frac{1}{m+n}$ because the ratio $\frac{\alpha_{is}}{\lfloor (n-1)\alpha_s + m\mu_s \rfloor/(n+m-1)}$ is bigger than 1. If his effort is smaller than the average effort exerted by others, his probability is smaller than $\frac{1}{m+n}$. The effort level of the non-expert white hat is still independent of the effort levels selected by other types of hackers (expert white hats and black hats), given that the other types do not allocate any effort to finding non-severe bugs. We focus,

therefore, only on the derivation of the effort levels of expert white hats and black hats selected at the equilibrium. Optimizing their objective functions, we obtain:

$$\frac{\partial R_{we}}{\partial \alpha_{is}} = \frac{K_s(t)(r_s + p_s)}{n + m} \left[\frac{1}{[(n-1)\alpha_s + m\mu_s]/(n + m - 1)} \right] - c_w \alpha_{is} = 0.$$

$$\frac{\partial R_b}{\partial \mu_{is}} = \frac{K_s(t)W}{n + m} \left[\frac{1}{[n\alpha_s + (m-1)\mu_s]/(n + m - 1)} \right] - c_b \mu_{is} = 0.$$

From the above conditions, it follows that $\frac{\partial \alpha_{is}}{\partial \mu_s} < 0$ and $\frac{\partial \mu_{is}}{\partial \alpha_s} < 0$. Hence, each expert white hat exerts less effort if the effort level of each black hat is higher at the equilibrium. And similarly, each black hat exerts less effort if the effort level of each expert white hat is higher at the equilibrium.

To characterize the symmetric equilibrium, we can evaluate the first order conditions at the symmetric equilibrium, where $\alpha_{is} = \alpha_s$ and $\mu_{is} = \mu_s$, to obtain the following system of two equations in α_s and μ_s as unknowns.

$$\frac{K_{s}(t)(r_{s}+p_{s})}{n+m} \left[\frac{1}{\lfloor (n-1)\alpha_{s}+m\mu_{s} \rfloor/(n+m-1)} \right] - c_{w}\alpha_{s} = 0$$

$$\frac{K_{s}(t)W}{n+m} \left[\frac{1}{\lfloor n\alpha_{s}+(m-1)\mu_{s} \rfloor/(n+m-1)} \right] - c_{b}\mu_{s} = 0.$$

We can use the implicit function theorem to investigate the effect of a higher bounty amount on the effort levels selected by each expert white hat and each black hat at the equilibrium. Total differentiation of the above system allows us to obtain the derivatives $\frac{\partial \alpha_s}{\partial p_a}$ and $\frac{\partial \mu_s}{\partial p_a}$ as follows:

$$\frac{d\alpha_{s}}{dp_{s}} = \left[\frac{K(t)W(m-1)}{(n+m)[n\alpha_{s}+(m-1)\mu_{s}]^{2}(n+m-1)} + c_{b}\right]\frac{c_{w}\alpha_{s}}{(R+P_{s})D} > 0.$$

$$\frac{d\mu_{s}}{dp_{s}} = -\left[\frac{K(t)m}{(n+m)[(n-1)\alpha_{s}+m\mu_{s}]^{2}(n+m-1)}\right]\frac{c_{w}\alpha_{s}}{D} < 0, \text{ where}$$

$$D = \frac{1}{(n+m-1)}\left[\frac{[K(s)]^{2}(r_{s}+p_{s})W}{(n+m)^{2}[n\alpha_{s}+(m-1)\mu_{s}]^{2}[(n-1)\alpha_{s}+m\mu_{s}]^{2}} + \frac{c_{w}K(t)W(m-1)}{(n+m)[n\alpha_{s}+(m-1)\mu_{s}]^{2}} + \frac{c_{b}K(t)(r_{s}+p_{s})(n-1)}{(n+m)[(n-1)\alpha_{s}+m\mu_{s}]^{2}}\right] + c_{w}c_{b}$$

Given the signs of the above derivatives, the results we obtain in the main text are strengthened. Specifically, in the main text we show that the bounty program reduces the probability that black hats are first to find severe bugs because the bounty incentivizes the white hats to allocate more effort to finding the bugs. With the new formulation, the bounty introduces a dual effect of concurrently incentivizing the white hats to increase efforts and the black hats to reduce efforts. This dual effect on effort is stronger than the single effect we obtain in the main text.

Appendix B: Case When eWHH Allocates Effort to Both Severe and Non-Severe Vulnerabilities

$$\begin{split} \alpha_{is} &> 0 \text{ and } \alpha_{ins} > 0 \text{ when } \left[\frac{\frac{c_{w}K_{ns}(t)P_{ns}}{n+l} - \frac{K_{s}(t)\left(R_{s}+P_{s}\right)}{n+m} \right] > 0. \\ \alpha_{is} &= \alpha_{s} = \frac{\left[\frac{K_{s}(t)\left(R_{s}+P_{s}\right)}{n+m} - \frac{K_{ns}(t)P_{ns}}{n+l} \right]}{(c_{w}-1)}, \quad \alpha_{ins} = \alpha_{ns} = \frac{\left[\frac{c_{w}K_{ns}(t)P_{ns}}{n+l} - \frac{K_{s}(t)\left(R_{s}+P_{s}\right)}{n+m} \right]}{(c_{w}-1)}. \\ \beta_{ins} &= \beta_{ns} = \frac{K_{ns}(t)P_{ns}}{n+l}. \\ \mu_{is} &= \mu_{s} = \frac{K_{s}(t)W}{c_{b}(n+m)}. \end{split}$$

Probability that expert white hat finds severe bug first: $\frac{1}{n+m} \left[1 + \left(\alpha_{is} - \frac{(n-1)\alpha_s + m\mu_s}{n+m-1} \right) \right] = \frac{1}{n+m} \left[1 + \frac{m}{n+m-1} \left\{ \frac{\left[\frac{K_s(t)(R_s+P_s)}{n+m} - \frac{K_w(t)P_m}{n+l} \right]}{(c_w-1)} - \frac{K_s(t)W}{c_b(n+m)} \right\} \right].$

Probability that expert white hat finds non-severe bug first: $\frac{1}{n+l} \left[1 + \left(\alpha_{ins} - \frac{(n-1)\alpha_{ns} + l\beta_{ns}}{n+l-1} \right) \right] = \frac{1}{n+l} \left[1 + \frac{l}{n+l-1} \left\{ \frac{\left[\frac{c_w K_{ns}(t)P_{ns}}{n+l} - \frac{K_s(t)(R_s + P_s)}{n+m} \right]}{(c_w - 1)} - \frac{K_{ns}(t)P_{ns}}{n+l} \right\} \right].$

Probability that non-expert white hat finds non-severe bug first:

$$\frac{1}{n+l} \left[1 + \left(\beta_{ins} - \frac{n\alpha_{ns} + (l-1)\beta_{ns}}{n+l-1} \right) \right] \\ \frac{1}{n+l} \left[1 + \frac{n}{n+l-1} \left(\beta_{ns} - \alpha_{ns} \right) \right] = \frac{1}{n+l} \left[1 + \frac{n}{n+l-1} \left\{ \frac{K_{ns}(t)P_{ns}}{n+l} - \frac{\left[\frac{c_w K_{ns}(t)P_{ns}}{n+l} - \frac{K_s(t)(R_s + P_s)}{n+l} \right] \right] \right]$$

Probability that black hat finds severe bug first : $\frac{1}{n+m} \left[1 + \left(\mu_{is} - \frac{n\alpha_s + (m-1)\mu_s}{n+m-1} \right) \right]$ (10)

$$\frac{1}{n+m} \left[1 + \frac{n}{n+m-1} \left(\mu_s - \alpha_s \right) \right] = \frac{1}{n+m} \left[1 + \frac{n}{n+m-1} \left\{ \frac{K_s(t)W}{c_b(n+m)} - \frac{\left[\frac{K_s(t)(R_s + P_s)}{n+m} - \frac{K_{ns}(t)P_{ns}}{n+l} \right]}{(c_w - 1)} \right\} \right].$$